

Roll No.

312

B. E. 1st Sem. (D. Scheme) Examination

May, 2007

MATHS - I

Paper : MATH-101-D

Time : Three hours]

[Maximum Marks : 100

Before answering the question, candidate should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions selecting at least two from each Part.

PART - A

1. (a) Using Taylor's, expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$. Hence find the value of $\sin 91^\circ$ correct to 4 decimal places. 10

(b) Find all the asymptotes of the curve.

$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$ 10

2. (a) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that

$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u.$ 10

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(b) Expand $\frac{(x+h)(y+k)}{x+h+y+k}$ in powers of h, k up to and inclusive of second degree terms. 10

3. (a) A rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 10

(b) Find the equation of circular cylinder with base as the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$. 10

4. (a) Test the convergence of the series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \quad 10$$

(b) Discuss the convergence of the series :

$$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots \quad 10$$

PART - B

5. (a) Evaluate the integral $\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$ by 10

changing the order of integration.

(b) Evaluate $\int_0^\pi \int_0^{a(1+\cos\theta)} r^2 \cos\theta \, dr \, d\theta$ 10

6. (a) Find the volume bounded by xy plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$. 10

(b) Show that :

$$\int_0^1 y^{q-1} \left(\log \frac{1}{y} \right)^{p-1} dy = \frac{\Gamma p}{q^p}; p > 0, q > 0. \quad 10$$

7. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. 10

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. 10

8. (a) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + z \hat{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. 10

(b) Verify divergence theorem for $\vec{F} = 4xz \hat{i} - y^2 \hat{j} + yz \hat{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 10

