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B.E. 2nd Semester Examination,

May - 2009

MATHEMATICS-II

Paper-Math-102-E

Time allowed : 3 hours]

[Maximum marks : 100

Note : Attempt five questions in all, selecting at least one question from each part.

Part-A

1. (a) Find the non-singular matrices P and Q such that PAQ is in the normal form for A. Hence find the rank of A, where

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & -2 & 0 & -2 \end{bmatrix} \quad 10$$

- (b) Find the sum and product of the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad 4$$

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- (c) Determine a, b, c so that the matrix A is orthogonal, where

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

2. (a) Test the consistency of the system of linear equations given below, if consistent find the solution :

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0$$

- (b) Find the inverse transformation of

$$y_1 = x_1 + x_2 + 2x_3$$

$$y_2 = 2x_1 + 5x_2 - 2x_3$$

$$y_3 = x_1 + 7x_2 - 7x_3$$

- (c) Verify Cayley-Hamilton Theorem for the matrix

$$\begin{bmatrix} 2 & -3 \\ 7 & -4 \end{bmatrix}$$

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Part-B

3. (a) Solve the differential equation given below :

$$(3x^2 y^4 + 2xy) dx + (2x^3 y^3 - x^2) dy = 0 \quad 10$$

- (b) A body is heated to 110°C and placed in air at 10°C . After 1 hour its temperature is 60°C . How much additional time is required for it to cool to 30°C ? 10

4. (a) Solve the differential equation

$$(D^2 - D)z = 2y + 1 + 4 \cos y + 2e^y,$$

where $D \equiv \frac{d}{dy}$ 10

- (b) Solve the following differential equation by the method of variation of parameters :

$$(D^2 + 4)y = 4 \sec^2 2x \quad 10$$

5. (a) Solve the following differential equation

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x) \quad 10$$

- (b) Determine the charge on the capacitor at any time $t > 0$ in a circuit in series having an emf given by $E(t) = 100 \sin 60 t$ V, a resistor of 2Ω , an inductor of 0.1 H and a capacitor of $\frac{1}{260}$ farads if the initial current and initial charge on the capacitor are both zero. Find the steady-state solution. 10

Part-C

6. (a) Write the following function $f(t)$ in terms of Heaviside's unit step function and hence find the Laplace transform of $f(t)$. 6
- (b) Find the Laplace inverse transform of the following :
- (i) $\cot^{-1} \left(\frac{s+a}{b} \right)$
- (ii) $\log \frac{s(s+1)}{(s^2+4)}$ 6
- (c) State and prove convolution theorem to find inverse Laplace Transform. 8

7. (a) Solve the following system of equations :

$$\frac{dx}{dt} - 6x + 3y = 8e^t;$$

$$\frac{dy}{dt} - 2x - y = 4e^t;$$

$$x(0) = -1, \quad y(0) = 0,$$

by Laplace transformation. 10

- (b) Form partial differential equation by eliminating the arbitrary function from

$$z = xf(ax + by) + g(ax + by) \quad 10$$

8. (a) Solve the partial differential equation,

$$(y^2 + z^2)p - xyq + zx = 0 \quad 6$$

- (b) Solve the following partial differential equations :

(i) $yp + xq + pq = 0$

(ii) $(p - q)(z - px - qy) = 1 \quad 5$

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- (c) Solve the following P.D.E. by the method of separation of variables :

$$u_{xx} = u_y + 2u, \quad \text{with } u(0, y) = 0,$$

$$\left(\frac{\partial u}{\partial x}\right)_{(0,y)} = 1 + e^{-2y} \quad 9$$

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