B.E. 1st Semester Examination,

May – 2009

MATHEMATICS–I

Paper–Math–I

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in all, selecting two questions from each part.*

Part–A

1. (a) Test the convergence or divergence of the series : 6

$$\sum \left[\sqrt{n^4+1}-\sqrt{n^4-1}\right]$$

(b) Discuss the convergence of the series 7

$$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\ldots\infty \ (x>0)$$

(c) State, with reasons, the values of x for which the series 7

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots\infty \text{ converges}$$

2. (a) Compute to four decimal places, the value of cos 32°, by use of Taylor's series. 6

(b) If ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be its focus, then show that ρ^2 varies as $(SP)^3$. 8

(c) Show that the assymptotes of the curve $x^2y^2 = a^2 (x^2 + y^2)$ form a square of side 2a. 6

3. (a) If $u = x^2 \tan^{-1} (y/x) - y^2 \tan^{-1} (x/y)$, evaluate

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$ 10

(b) If $f(x, y) = \tan^{-1}(xy)$, compute $f(0.9, -1.2)$ approximately. 10

4. (a) Find the maximum and minimum distances from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$ 10

(b) Evaluate $\int_0^{\alpha} \frac{\log(1+\alpha x)}{1+x^2} dx$ 10

Part–B

5. (a) Find the volume of solid formed by revolving a loop of the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line. 10

(b) Evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$ by changing the order of integration. 10

6. (a) Evaluate $\iiint (x + y + z)\, dx\, dy\, dz$ over the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. 10

(b) Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. 10

7. (a) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$ 10

(b) If v_1 and v_2 be the vectors joining the fixed points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point (x, y, z), prove that

$$\text{Curl}\,(v_1 \times v_2) = 2\,(v_1 - v_2)$$ 10

8. (a) Verify Stoke's theorem for $F = (x^2 + y^2)\,\hat{i} - 2xy\,\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. 10

(b) Using divergence theorem, evaluate $\int_S \overline{R} . N\, ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. 10