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B. E. 2nd Sem. Examination May, 2007

MATHEMATICS – II

Paper : MATH-102-E

Time : Three hours] *[Maximum Marks : 100*

Before answering the question, candidate should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting at least ***one*** question from each Section.

SECTION – A

1. (a) Find the rank of :

$$\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

by reducing it to normal form. 10

(b) Find the inverse of :

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

using elementary transformations. 10

2. (a) Solve the equations by matrix method :

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

10

(b) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & -2 \end{bmatrix}$$ and hence find A^{-1}. 10

SECTION – B

3. (a) Solve the D.E :

$$(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0.$$

10

(b) Solve the D.E :

$$\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)\,dy = 0.$$ 10

4. (a) The charge Q on the plate of a condenser of capacity C charged through a resistance R by a steady voltage V satisfies the differential eqn.

$$R\frac{dQ}{dt} + \frac{Q}{C} = V$$

If Q = 0 at t = 0, show that :

$$Q = CV\left(1 - e^{-\frac{t}{RC}}\right)$$ 10

(b) Solve the D.E :

$$(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2).$$ 10

5. (a) Solve by method of variation of parameters :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x.$$ 10

(b) Solve :

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$ 10

SECTION – C

6. (a) Find the Laplace transform of :

(i) $t \sin^3 t$

(ii) $e^{-t} \frac{\sin t}{t}$ 10

(b) Find the inverse Laplace transform of :

$$\frac{s^2}{(s+1)^3}$$ 10

7. (a) Find the Laplace transform of function f(t) given by :

$$f(t) = \begin{cases} t & 0 < t < c \\ 2c - t & c < t < 2c \end{cases}$$ 10

(b) Form the PDE by eliminating the arbitrary functions from :

$$z = x f(x+t) + g(x+t)$$ 10

8. (a) Solve :

$$z^2 (p^2 x^2 + q^2) = 1$$ 10

(b) Determine the solution of one-dimensional heat equation :

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(0, t) = 0 = u(l, t) \quad (t > 0)$

and $u(x, 0) = x$, l being length of the bar. 10