

Roll No.

1202

B. E. 1st Sem. (E. Scheme)

Examination – May, 2008

MATHEMATICS - I

Paper : Math - 1

Time : Three hours]

[Maximum Marks : 100

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting two questions from each Part.

PART - A

1. (a) Test the convergence or divergence of the series : 6

$$\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \dots$$

- (b) Discuss the convergence of the series : 7

$$\frac{x}{1} + \frac{1}{2} \frac{x^3}{3} + \frac{1.3}{2.4} \frac{x^5}{5} + \frac{1.3.5}{2.4.6} \frac{x^9}{7} + \dots (x > 0)$$

- (c) Using the integral test, discuss the convergence of the series : 7

$$\sum \frac{1}{(n+1)^2}$$

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2. (a) Expand $e^{\sin x}$ by Maclaurin's series up to term containing x^4 . 6

(b) Show that the radius of convergence at any point of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos \frac{\theta}{2}$. 8

(b) Find the asymptotes of the curve : 6

$$x^3 + 3x^2y - 4y^3 - x + y + z = 0$$

3. (a) If $u = f(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$ show that : 10

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$$

(b) Prove that : 10

$$\int \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a) \text{ when } a \geq 0.$$

4. (a) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree. 10

(b) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid : 10

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

PART - B

5. (a) Find the triple integration, the volume in the positive octant by the co-ordinate planes and the plane $x + 2y + 3z = 4$. 10

- (b) Evaluate by changing the order of integration : 10

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$$

6. (a) Evaluate the integral by changing into polar-coordinates : 10

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dx dy$$

- (b) Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of gamma

functions and evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$. 10

7. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, prove that $\text{div} (r^n \vec{r}) = (n+3)r^n$. 10

- (b) Show that : 10

$$\text{curl} (\text{curl } \vec{V}) = \text{grad div } \vec{V} - \nabla^2 \vec{V}$$

8. (a) If $\vec{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$, then evaluate

$\iiint_V \vec{F} \cdot dV$, where V is bounded by the planes
 $x=0, y=0, z=0$ and $2x+2y+z=4$. 10

(b) Verify divergence theorem for:

$$\vec{F} = 4x^2\vec{i} - 2y^2\vec{j} + z^2\vec{k}$$

taken over the region bounded by the cylinder: 10

$$x^2 + y^2 = 4, z=0, z=3$$