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1211

B. E. 2nd Sem.

Examination – May, 2008

MATHEMATICS - II

Paper : Math - 102 - E

Time : Three hours] *[Maximum Marks : 100*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *one* question from each Part.

PART – A

1. (a) For the matrix :

$$A=\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix},$$

find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A.

(b) Discuss the consistency of the system of equations :

$2x-3y+6z-5w=3,\ y-4z+w=1,$

P. T. O.

$$4x - 5y + 8z - 9w = \lambda$$

for various values of λ, if consistent, find the solution.

2. (a) (i) Prove that the sum of the eigen values of a matrix A is the sum of the elements of the principal diagonal.

(ii) Find the sum and product of the eigen values of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 5 & 6 \\ 7 & 4 & 3 & 2 \\ 4 & 3 & 0 & 5 \end{bmatrix}$$

(b) Verify that the following set of vectors in $\mathbb{R}^3$ is linearly dependent : (1, 0, 1), (1, 1, 1), (1, 1, 2) and (1, 2, 1) : Also find the number of linearly independent vectors.

PART – B

3. (a) Solve :

$$(x^2y^2 + xy + 1)\, ydx + (x^2y^2 - xy + 1)\, xdy = 0.$$

(b) Solve for the current $I(t)$ in an RL circuit if R = 2 ohms, L = 25 henrys and E(t) = Ae^{-t} with $A > 0$, as a constant and I (0) = 0.

4. (a) Solve the differential equation :

$$(D^2 - 4)y = x\sinh(x)$$

(b) Solve the differential equations :

$$(x^2D^2 - xD - 3)\,y = x^2(\log x)^2$$

5. (a) Solve the differential equation by the method of variations of parameters :

$$y'' + y = \sec^2 x$$

(b) Determine the current I(t) in an LCR circuit with e. m. f. E(t) = $E_0 \sin wt$, in case the circuit is tuned

to resonance so that $w^2 = 1/LC$ and R/L is so small that second and higher degree terms can be rejected. Assuming that at t = 0, I (0) = I' (0) = 0.

PART – C

6. (a) Find the Laplace transform of each of the following :

(i) $e^{4t} \sin 2t \cos t$,

(ii) $\sin h(t) \cos^2 t$,

(iii) $e^{-t} \cos^2 t$.

(b) Solve the simultaneous differenial equations using Laplace transforms :

$$x^1(t) + y^1(t) + x(t) = -e^{-t}, \; x^1(t) + 2y^1(t) + 2x(t) + 2y(t) = 0$$

where $x(0) = -1,\; y(0) = 1$.

7. (a) Find the inverse Laplace transform of :

(i) $\frac{3s+1}{s^2(s^2+4)}e^{-3s}$

(ii) $\tan^{-1}\left(\frac{2}{s^2}\right)$

(b) Solve the partial differential equation :

$$x^2(y-z)p+y^2(z-x)q=z^2(x-y)$$

8. (a) Solve the partial differential equation :

$z^2 = pqxy$

(b) Find the temperature in a thin metal rod of length L, with both the ends insulated (so that there is no passage of heat through the ends) and with initial temperature in the rod $\sin(\pi x/L)$.